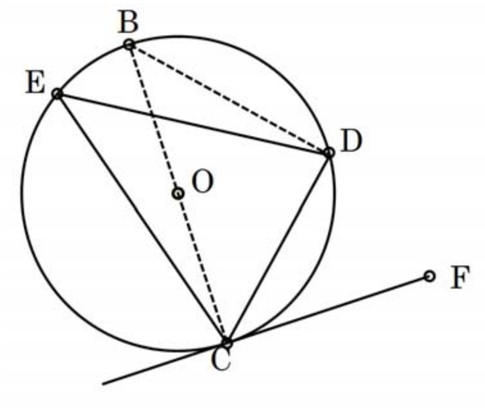
Activity 18 More circle theorems

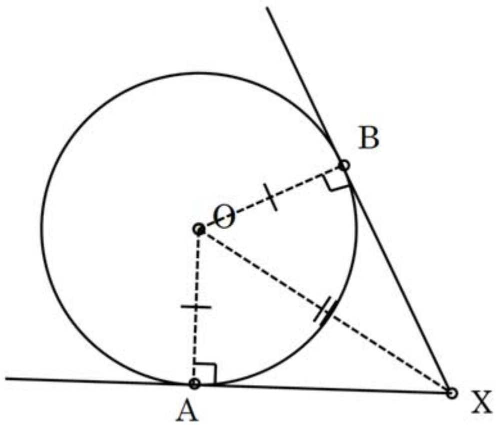
1. The angle between a tangent and a chord is equal to the angle in the alternate segment.



Let $\angle DCF = a$

$\angle BCF = 90°$	Tangent is perpendicular to radius at point of contact
$\angle BCD = 90° - a$	Complementary angles
$\angle BDC = 90°$	Angle in a semi-circle
$\angle DBC = a$	Angle sum in a triangle is 180°
$\angle DEC = a$	Angles in same segment

2. Tangents to a circle drawn from a common point have the same length.



$\angle OAX = 90°$	Tangent is perpendicular to radius at point of contact
$\angle OBX = 90°$	Tangent is perpendicular to radius at point of contact
OA=OB	Radii of the same circle

OX is common to ΔOAX and ΔOBX

$\Delta OAX \cong \Delta OBX$	R.H.S
AX=BX	Corresponding sides of congruent triangles

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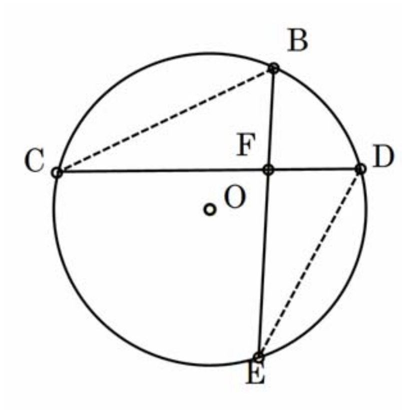
3. When two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord.



$\angle$BCD =$\angle$BED Angles in the same segment theorem.

$\angle$CBE =$\angle$CDE Angles in the same segment theorem.

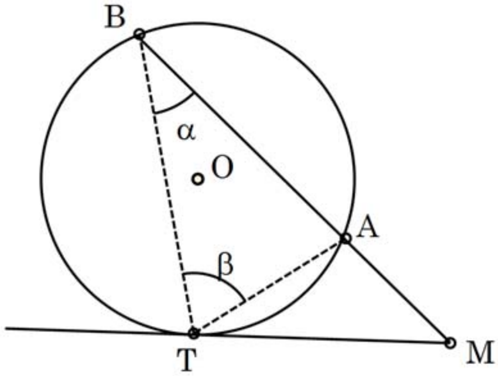
ΔCBF is similar to ΔEDF Equal angles in both triangles

CF:EF = BF:DF Corresponding sides of similar triangles

$$\frac{CF}{EF} = \frac{BF}{DF}$$

$$CF \times DF = BF \times EF$$

4. When a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant. ($AM \times BM = TM^2$)



Let $\angle$TBA = α

$\angle$ATM = α Angles in alternate segment theorem.

Let $\angle$TBA = β

$\angle$BAT =180 –($\alpha + \beta$) Angle sum in a triangle

$\angle$TAM = $\alpha + \beta$ Angles on a straight line

$\angle$BTM = $\angle$BTA + $\angle$ATM = $\alpha + \beta$

ΔATM is similar to ΔTBM Equal angles in both triangles

TM:BM = AM:TM Corresponding sides of similar triangles

$$\frac{TM}{BM} = \frac{AM}{TM}$$

$$TM^2 = AM \times BM$$

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